Optum Concrete Solution

Verification: Beam bending

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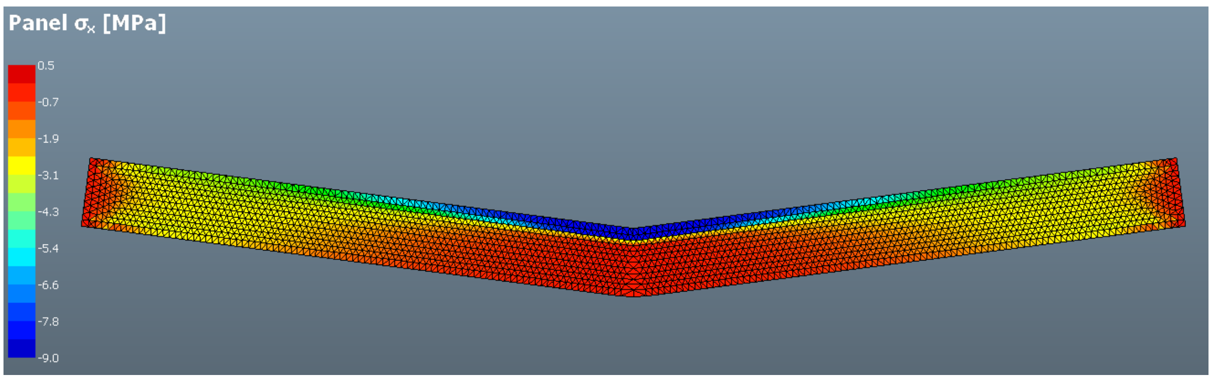
Contents

1 Introduction

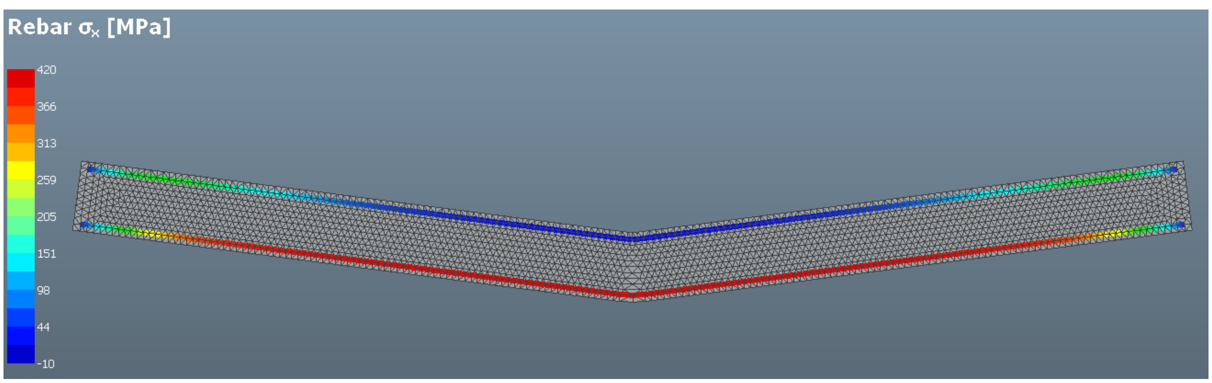
This document verifies the plane stress elements available in Optum Concrete Soltuion (OptumCS). For this purpose a beam with simple supports is considered. The beam fails in bending and the exact capacity can be calculated by assuming a plastic distribution of normal forces.

2 Benchmark A: Beam with rebars and stirrups

The plastic bending capacity is given as 67.538 kNm corresponding to a **line load of 9.354 kN/m**.



(a) Concrete stress



(b) Rebar stress

Figure 1: Horizontal normal stresses, σ_x, and collapse model of the beam using 4000 mixed elements: Limit load $p = 9.282$ kN/m.

	Lower bound		Mixed		Upper bound	
nel	p [kN/m]	Error	p [kN/m]	Error	p [kN/m]	Error
1000	9.136	2.33 %	9.141	2.28 %	9.498	1.54 %
2000	9.260	1.01 %	9.262	0.98 %	9.421	0.71 %
4000	9.282	0.78 %	9.282	0.77 %	9.400	0.49 %

The results are compiled in the table above. All elements converge monotonously towards the exact solution with the upper bound being the most accurate in this case.

2.1 Analytical solution

The considered beam has stirrups and longitudinal rebars in the top and bottom. The following parameters are used:

$f_{ck} = 25.0$ MPa	Characteristic concrete compressive strength
$\nu = 0.5$	Effectiveness factor
$\gamma_c = 1.4$	Partial coefficient for concrete
$f_{cd} = 8.93 \text{ MPa} = \frac{\nu}{\gamma_c} f_{ck}$	Design concrete compressive strength
$f_{yd} = 416.7$ MPa	Design reinforcement tensile strength
$A_t = 402 \text{ mm}^2$	Reinforcement area in the top of the beam
$A_b = 402 \text{ mm}^2$	Reinforcement area in the bottom of the beam
$L = 7600$ mm	Span of the beam

Cross section of the beam is shown below. The maximum tension force in the bottom reinforcement

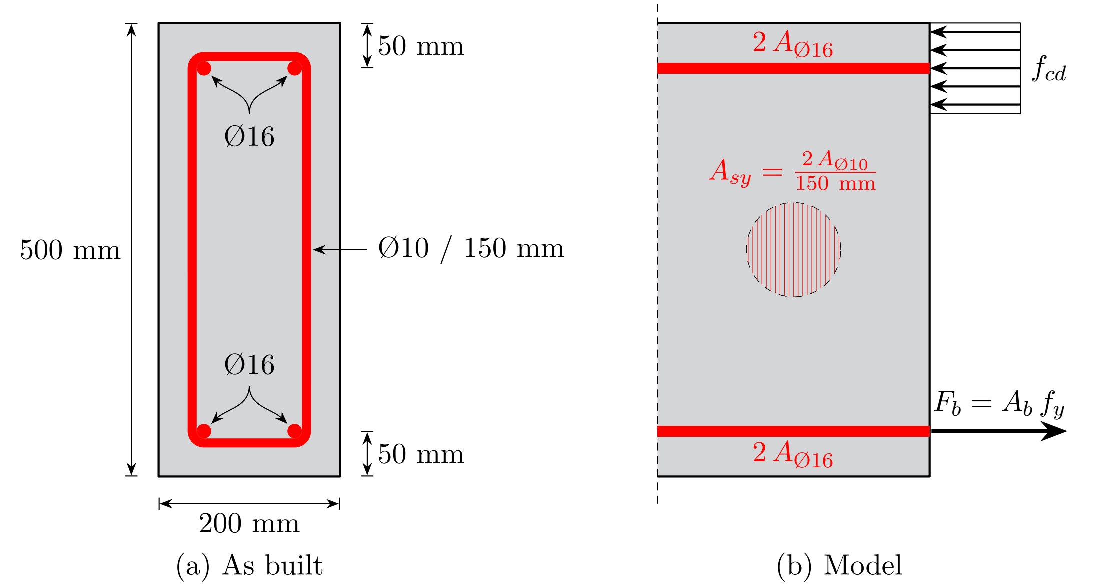


Figure 2: Cross section of the beam (a), reinforcement in model and plastic stress distribution (b).

is:

$$F_b = 167.6 \text{ kN}$$

which gives a compression zone with a height of 93.8 mm. The distance from the bottom rebars to the centre of the compression zone is $d = 403.1$ mm, thus, the plastic moment capacity becomes:

$$M_{Rd} = d\,F_b = 67.538 \text{ kN/m}$$

With a span of 7600 mm, this corresponds to load carrying capacity of 9.354 kN/m.

3 Benchmark B: Beam with mesh reinforcement

For this example, the plastic bending capacity is given as 18.047 kNm corresponding to a **line load of 2.500 kN/m**.

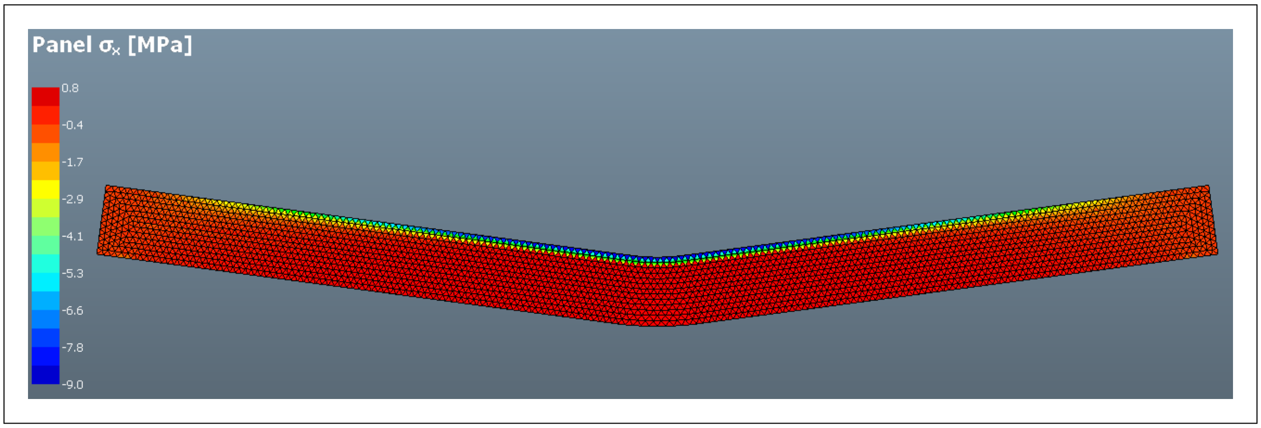


Figure 3: Horizontal normal stress, σ_x, and collapse model of the beam using 4000 upper bound elements: Limit load $p = 2.513$ kN/m.

	Lower bound		Mixed		Upper bound	
nel	p [kN/m]	Error	p [kN/m]	Error	p [kN/m]	Error
1000	2.381	4.77 %	2.390	4.38 %	2.598	3.95 %
2000	2.433	2.66 %	2.434	2.63 %	2.544	1.77 %
4000	2.478	0.86 %	2.478	0.86 %	2.513	0.54 %

The results are compiled in the table above. All elements converge monotonously towards the exact solution with the upper bound being the most accurate in this case.

3.1 Analytical solution

The considered beam has mesh reinforcement horizontally and vertically. The following parameters are used:

$f_{ck} = 25.0$ MPa	Characteristic concrete compressive strength
$\nu = 0.5$	Effectiveness factor
$\gamma_c = 1.4$	Partial coefficient for concrete
$f_{cd} = 8.93 \text{ MPa} = \frac{\nu}{\gamma_c} f_{ck}$	Design concrete compressive strength
$f_{yd} = 416.7$ MPa	Design reinforcement tensile strength
$A_{sx} = A_{sy} = 377.0 \text{ mm}^2$	Reinforcement area per unit length
$L = 7600$ mm	Span of the beam

The reinforcement area corresponds to two layers of Ø6 rebars per 150 mm. Cross section of the beam is shown below. The maximum tension force per unit length in the horizontal reinforcement

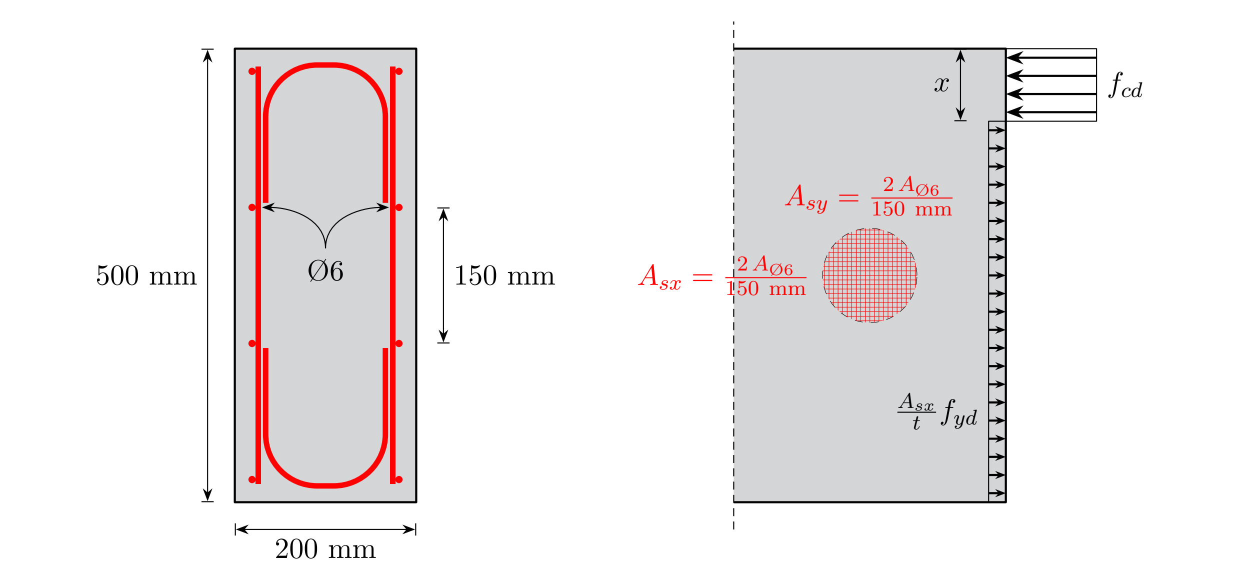


Figure 4: Cross section of the beam (a), reinforcement in model and plastic stress distribution (b).

is:

$$f_s = A_{sx} f_{yd} = 157.1\ \text{kN/m}$$

The height of the compression zone is calculated to be $x = 40.4$ mm, thus, the plastic moment capacity becomes:

$$M_{Rd} = f_s \left(h - x\right) \frac{h}{2} = 18.047\ \text{kN/m}$$

With a span of 7600 mm, this corresponds to load carrying capacity of 2.500 kN/m.

4 Modelling in OptumCS

The beam is supported compression-only supports at either end. The total length of the beam is 8 metres, and the support lengths are 200 mm in either end leaving a span of 7600 mm. Fig. 5 and Fig. 6 show the real reinforcement layout of the two beams, how it is modelled analytically, and how the model looks in OptumCS. The specific reinforcement diameters and strengths are given in the previous sections.

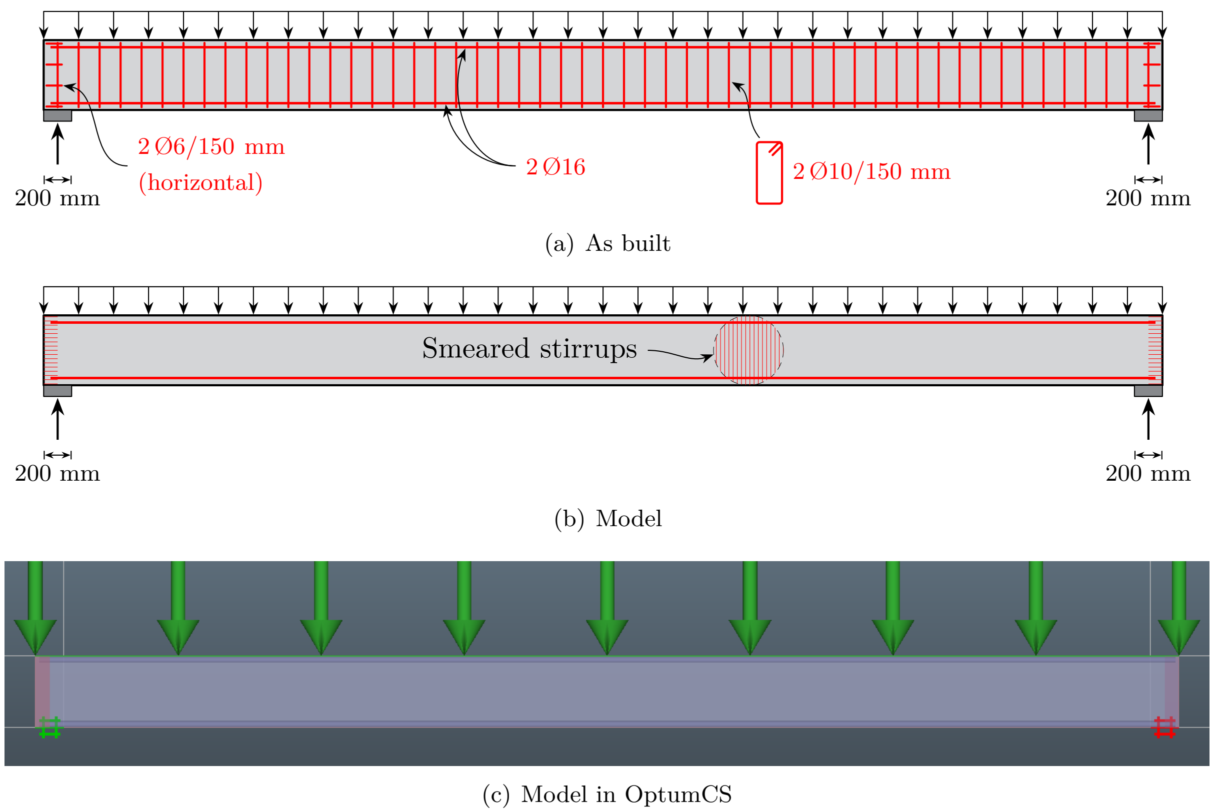


Figure 5: Benchmark A: Reinforcement layout in reality and model, supports and load.

Fig. 5 shows horizontally placed stirrups near the beam ends above the supports. These are placed solely for numerically reasons: Having unreinforced elements with a tensile strength of zero near the beam ends affects the collapse mode of the lower bound elements. Adding the horizontal stirrups mitigates this. Alternatively, a minuscule tensile strength could be introduced in the concrete, however, this would also affect the bending capacity, albeit insignificantly.

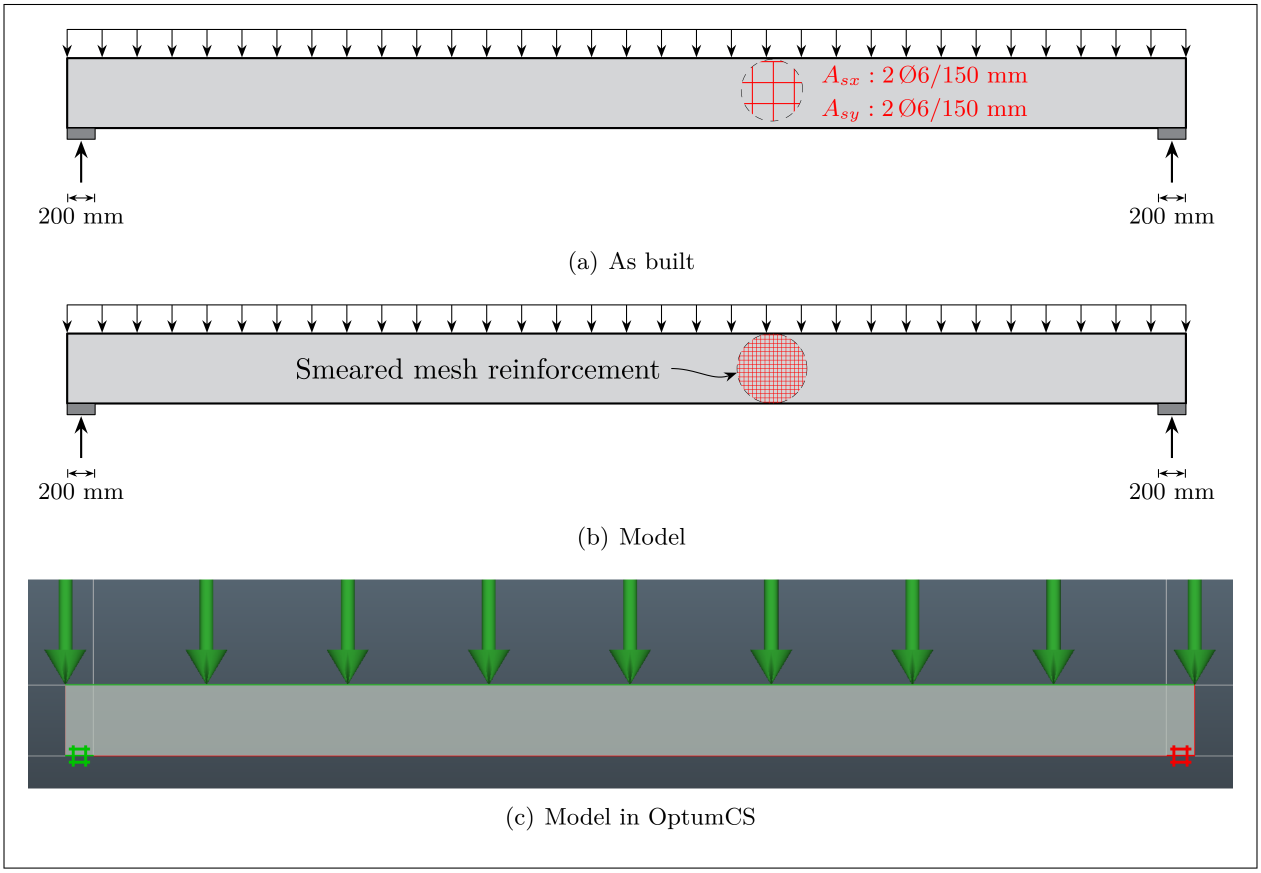


Figure 6: Benchmark B: Reinforcement layout in reality and model, supports and load.

4.1 Finite elements

Optum Concrete Solution has three types of elements available: Lower bound, upper bound, and mixed.

The lower bound element will compute a limit load lower than the exact solution and it ensures a statically admissible and safe stress field, i.e. a stress field that satisfies equilibrium and the yield function in every point. The lower bound element is in general the least accurate, but it has the advantage that it is always on the safe side.

The upper bound element will compute a limit load higher than the exact solution. It ensures a kinematically admissible solution, i.e. the strain-displacement relations are satisfied in every point. Moreover, the flow rule is likewise satisfied in every point.

The mixed element used in OptumCS is a relaxed version of the lower bound element. This increases the accuracy and numerical robustness of the element substantially, but it no longer ensures a lower bound solution. It will, nevertheless, typically approach the exact limit load from below. The mixed element is the default and recommended element in OptumCS.

4.2 Materials

Property	Value
Wall block	
Name	
Description	
Color	[186, 186, 186] (255)
Thickness [mm]	200
Concrete	C25
Steel	S500
Weight [kN/m³]	0.00
Horizontal Reinforcement	
Diameter [mm]	6.00
Layers	2
Spacing [mm]	150
Anchorage [m...	0
Vertical Reinforcement	
Diameter [mm]	6.00
Layers	2
Spacing [mm]	150
Anchorage [m...	0

(a) Wall block

Property	Value
Reinforcement	
Name	Bjælke
Description	
Color	[125, 125, 179] (255)
Coverage [mm]	50
Beam Bottom / Column Left Rebar	
Steel	S500
Diameter [mm]	16.00
Number of bars	2
Beam Top / Column Right Rebar	
Steel	S500
Diameter [mm]	16.00
Number of bars	2
Stirrup	
Steel	S500
Diameter [mm]	10.00
Layers	2
Spacing [mm]	150

(b) Reinforcement block

Figure 7: Material for the beam.

OptumCS uses a composite material model for reinforced concrete. The mesh reinforcement and stirrups are treated as smeared, i.e. as a layer of steel, instead of as individual bars.

- **Concrete:**
  - The Mohr-Coulomb yield criterion is used with a friction angle of 37°.
  - A tension cut-off limits the tensile stress in the concrete. The tensile strength is set to zero.
  - The wall is defined with C25 concrete. With an effectiveness factor $\nu = 0.5$ and a partial coefficient of $\gamma_c = 1.4$, the design compressive strength is 8.93 MPa.
- **Mesh reinforcement:**
  - The reinforcement only carries tension. Compression and shear is carried by the concrete.
  - A simple tension limit is used as the yield criterion for the reinforcement.
  - The reinforcement is treated as fully anchored, i.e. the full strength of the reinforcement can be utilised in every point.
- **Discrete reinforcement:**
  - Modelled using bar elements.

  - A simple tension limit is used as the yield criterion for the reinforcement.
  - Discrete reinforcement is modelled with an anchorage length of 40 times the diameter, i.e. 40 times the diameter is required to build up to the yield strength of the bar.
  - The normal force at free bar ends is required to be zero unless an anchorage force is specified.

- **Building blocks**
  - A wall block is used to define the beam and the concrete.
  - The rebars and stirrups are defined in the reinforcement block, which overrides the reinforcement of the wall block.
  - The stirrups of a reinforcement block is treated as fully anchored, and the full strength can be utilized everywhere.

- Partial coefficient γ_c for concrete is set to 1.4.
- Partial coefficient γ_s for steel is set to 1.2.